Spiral plat avec courbes terminales externe et interne

Anisochronisme en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5 \times 10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d_V := 1.1 \cdot mm \quad d_B := 1.312 \cdot d1_{sp} \quad p_{sp} = 0.135\,mm \quad n_{sp} := \dfrac{d2_{sp} - d_B}{2 \cdot p_{sp}}$

$L := \pi \cdot \dfrac{n_{sp}}{2} \cdot \left(d2_{sp} + d_B\right) \quad L = 10.674\,cm \quad \psi_0 := 2 \cdot \pi \cdot n_{sp} \quad \psi_0 = 4.102 \times 10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5 \cdot d2_{sp} \quad z_A := r_A \cdot e^{i \cdot \alpha_A}$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2 \cdot \pi} \quad r_s(\alpha) := r_A - a \cdot \left(\alpha - \alpha_A\right) \quad x_{0s}(\alpha) := r_s(\alpha) \cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha) \cdot sin(\alpha)$

$s(\alpha) := \dfrac{1}{2 \cdot a} \cdot \left(r_A^2 - r_s(\alpha)^2\right) \quad s(\alpha) := r_A \cdot \left(\alpha - \alpha_A\right) - \dfrac{a}{2} \cdot \left(\alpha - \alpha_A\right)^2$

Courbe terminale externe

$r_{t1} := 0.8 \quad r_{t1} := racine\left[\left(2 \cdot r_{t1} - 1\right)^4 - 4 \cdot \left(1 - r_{t1}\right)^4 - \pi^2 \cdot r_{t1}^2 \cdot \left(1 - r_{t1}\right)^2, r_{t1}\right] \cdot r_A \quad r_{t1} = 0.832\,r_A$

$r_{t2} := 2 \cdot r_{t1} - r_A \quad r_{t2} = 0.665\,r_A \quad \beta_0 := arctan\left[\dfrac{\pi \cdot r_{t1}}{2 \cdot \left(r_A - r_{t1}\right)}\right] \quad \beta_0 = 82.695\,deg \quad l_t := r_{t2} \cdot \beta_0 + \pi \cdot r_{t1}$

$x_{0t1}\left(\alpha_t\right) := -r_A + r_{t1} \cdot \left(1 + cos\left(\alpha_t\right)\right) \quad y_{0t1}\left(\alpha_t\right) := r_{t1} \cdot sin\left(\alpha_t\right)$

$x_{0t2}\left(\beta_t\right) := r_{t2} \cdot cos\left(\beta_t\right) \quad y_{0t2}\left(\beta_t\right) := r_{t2} \cdot sin\left(\beta_t\right)$

Courbe terminale interne $\quad \alpha_B := mod\left(\psi_0 + \pi, 2 \cdot \pi\right) \quad \alpha_B = 322.4\,deg \quad r_B := 0.5 \cdot d_B$

$\beta := 121 \cdot deg \quad \beta'_0 := racine\left[\beta \cdot \left(\sqrt{2} \cdot sin(\beta) - 1\right) + sin(\beta) \cdot cos(\beta), \beta\right] \quad \beta'_0 = 121.21\,deg$

$r_{t'} := \dfrac{r_B}{\sqrt{2} \cdot sin\left(\beta'_0\right)} \quad r_{t'} = 0.827\,r_B \quad x_{0t}\left(\alpha_t\right) := -r_B + r_{t'} \cdot \left(1 + cos\left(\alpha_t\right)\right) \quad y_{0t}\left(\alpha_t\right) := r_{t'} \cdot sin\left(\alpha_t\right)$

$x_{0t'}\left(\alpha_{t'}\right) := \left[r_B + r_{t'} \cdot \left(-1 + cos\left(\alpha_{t'}\right)\right)\right] cos\left(\alpha_B\right) - r_{t'} \cdot sin\left(\alpha_{t'}\right) \cdot sin\left(\alpha_B\right) \quad l'_t := r_{t'} \cdot 2 \cdot \beta'_0$

$y_{0t'}\left(\alpha_{t'}\right) := \left[r_B + r_{t'} \cdot \left(-1 + cos\left(\alpha_{t'}\right)\right)\right] sin\left(\alpha_B\right) + r_{t'} \cdot sin\left(\alpha_{t'}\right) \cdot cos\left(\alpha_B\right) \quad L_t := l_t + L + l'_t$

Position des goupilles de raquettes $\quad r_{GR} := r_{t2} \quad \alpha_{GR} := -\beta_0 \quad \alpha_{GR} = -82.695\,deg$

$x_{GR} := x_{0t2}\left(\alpha_{GR}\right) \quad y_{GR} := y_{0t2}\left(\alpha_{GR}\right)$

Position du point d'attache à la virole $\quad r_V := \sqrt{x_{0t'}\left(2 \cdot \beta'_0\right)^2 + y_{0t'}\left(2 \cdot \beta'_0\right)^2} \quad \alpha_V(\theta) := Atan\left(x_{0t'}\left(2 \cdot \beta'_0\right), y_{0t'}\left(2 \cdot \beta'_0\right)\right) + \theta$

$r_V = 0.55\,mm \quad \alpha_V(0) = 216.447\,deg \quad x_V(\theta) := r_V \cdot cos\left(\alpha_V(\theta)\right) \quad y_V(\theta) := r_V \cdot sin\left(\alpha_V(\theta)\right)$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$I_{33} := I_{f\_rect}(ép, ha)$$

Déplacement de la virole libre

Contribution du spiral sans ses courbes terminales

$$s_s(\alpha) := s(\alpha) + l_t \qquad z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha) \qquad f_s(\theta,\alpha) := i\cdot\theta\cdot\exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$$

$$\Delta_s(\theta) := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot f_s(\theta,\alpha)\cdot r_s(\alpha)\,d\alpha \qquad \Delta_s(\theta_0) = 0.164 + 0.043i\,mm$$

Approximation $\quad OA := r_A\cdot e^{i\cdot\pi} \qquad OB := r_B\cdot e^{i\cdot(\pi+\psi_0)} \qquad f'_s(\theta,\alpha) := \frac{-\theta^2}{L_t}\cdot r_s(\alpha)\cdot\exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$$\Delta_{as}(\theta) := \frac{1}{L_t}\cdot\left[\left[(i\cdot r_A + 2\cdot a)\cdot f_s(\theta,\pi) - r_A\cdot f'_s(\theta,\pi)\right]\cdot OA + \left[-(i\cdot r_B + 2\cdot a)\cdot f_s(\theta,\pi+\psi_0) + r_B\cdot f'_s(\theta,\pi+\psi_0)\right]\cdot OB\right]$$

$$\Delta_{as}(\theta) := \frac{\theta}{L_t}\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left[(-r_A + i\cdot 2\cdot a)\cdot OA - (-r_B + i\cdot 2\cdot a)\cdot e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot OB\right] + \frac{\theta^2}{L_t^{\,2}}\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(r_A^{\,2}\cdot OA - r_B^{\,2}\cdot e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot OB\right)$$

$$\Delta_{as}(\theta_0) = 0.163 + 0.042i\,mm$$

Contribution de la courbe terminale externe

$$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i\cdot y_{0t1}(\alpha_t) \qquad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i\cdot y_{0t2}(\beta_t)$$

$$\Delta_{t2}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_{t2}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot\exp\left[i\cdot\frac{\theta}{L_t}\cdot\left[r_{t2}\cdot(\beta_0+\beta_t)\right]\right]d\beta_t \qquad \Delta_{t2}(\theta_0) = 0.074 + 0.092i\,mm$$

$$\Delta_{t1}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_{t1}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot\exp\left[i\cdot\frac{\theta}{L_t}\cdot(r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t)\right]d\alpha_t \qquad \Delta_{t1}(\theta_0) = -0.239 - 0.141i\,mm$$

$$\Delta_t(\theta) := \Delta_{t1}(\theta) + \Delta_{t2}(\theta) \qquad \Delta_t(\theta_0) = -0.164 - 0.049i\,mm$$

Approximations

$$s_{t2}(\beta_t) := r_{t2}\cdot(\beta_0+\beta_t) \qquad f_{t2}(\theta,\beta_t) := i\cdot\theta\cdot\exp\left(i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}\right) \qquad f'_{t2}(\theta,\beta_t) := \frac{-\theta^2}{L_t}\cdot r_{t2}\cdot\exp\left(i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}\right)$$

$$l_{t2} := r_{t2}\cdot\beta_0 \qquad Og_{12} := \frac{r_{t2}}{l_{t2}}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\,d\beta_t \qquad Og_{22} := \frac{2\cdot r_{t2}}{l_{t2}^{\,2}}\cdot\int_{-\beta_0}^{0} r_{t2}\cdot\beta_t\cdot z_{0t2}(\beta_t)\,d\beta_t$$

$$\Delta_{at2}(\theta) := \frac{1}{L_t}\cdot\left(l_{t2}\cdot f_{t2}(\theta,0)\cdot Og_{12} + f'_{t2}(\theta,0)\cdot\frac{l_{t2}^{\,2}}{2\cdot r_{t2}}\cdot Og_{22}\right) \qquad \Delta_{at2}(\theta_0) = 0.074 + 0.092i\,mm$$

$$s_{t1}(\alpha_t) := (r_{t2}\cdot\beta_0 + r_{t1}\cdot\alpha_t) \qquad f_{t1}(\theta,\alpha_t) := \mathrm{i}\cdot\theta\cdot exp\left(\mathrm{i}\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right) \qquad f'_{t1}(\theta,\alpha_t) := \frac{-\theta^2}{L_t}\cdot r_{t1}\cdot exp\left(\mathrm{i}\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right)$$

$$l_{t1} := r_{t1}\cdot\pi \qquad \boldsymbol{Og_{11}} := \frac{r_{t1}}{l_{t1}}\cdot\int_0^{\pi} z_{0t1}(\alpha_t)\,d\alpha_t \qquad \boldsymbol{Og_{21}} := \frac{2\cdot r_{t1}}{{l_{t1}}^2}\cdot\int_0^{\pi} r_{t1}\cdot\alpha_t\cdot z_{0t1}(\alpha_t)\,d\alpha_t$$

$$\boldsymbol{\Delta_{at1}}(\theta) := \frac{1}{L_t}\cdot\left(l_{t1}\cdot f_{t1}(\theta,0)\cdot\boldsymbol{Og_{11}} + f'_{t1}(\theta,0)\cdot\frac{{l_{t1}}^2}{2\cdot r_{t1}}\cdot\boldsymbol{Og_{21}}\right) \qquad \boldsymbol{\Delta_{at1}}(\theta_0) = -0.24 - 0.145\mathrm{i}\,mm$$

$$\boldsymbol{Og_1} := \frac{1}{l_t}\cdot(l_{t1}\cdot\boldsymbol{Og_{11}} + l_{t2}\cdot\boldsymbol{Og_{12}}) \qquad \boldsymbol{Og_1} = 0.632\mathrm{i}\,mm$$

$$\boldsymbol{\Delta_{at}}(\theta) := \boldsymbol{\Delta_{at1}}(\theta) + \boldsymbol{\Delta_{at2}}(\theta) \qquad \boldsymbol{\Delta_{at}}(\theta_0) = -0.166 - 0.053\mathrm{i}\,mm$$

Contribution de la courbe terminale interne

$$z_{0t'}(\alpha_{t'}) := x_{0t'}(\alpha_{t'}) + \mathrm{i}\cdot y_{0t'}(\alpha_{t'}) \qquad s_{t'}(\alpha_{t'}) := r_{t'}\cdot\alpha_{t'} + L + l_t \qquad \alpha_B = 322.4\,deg$$

$$\boldsymbol{\Delta_{t'}}(\theta) := \frac{\mathrm{i}\cdot\theta}{L_t}\cdot\int_0^{2\cdot\beta'_0} z_{0t'}(\alpha_{t'})\cdot exp\left(\mathrm{i}\cdot\frac{\theta}{L_t}\cdot s_{t'}(\alpha_{t'})\right)\cdot r_{t'}\,d\alpha_{t'} \qquad \boldsymbol{\Delta_{t'}}(\theta_0) = 0.013 + 0.015\mathrm{i}\,mm$$

Approximations

$$s(\alpha_{t'}) := L + l_t + r_{t'}\cdot\alpha_{t'} \qquad f_{t'}(\theta,\alpha_{t'}) := \mathrm{i}\cdot\theta\cdot exp\left(\mathrm{i}\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}\right) \qquad f'_{t'}(\theta,\alpha_{t'}) := \frac{-\theta^2}{L_t}\cdot r_{t'}\cdot exp\left(\mathrm{i}\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}\right)$$

$$\boldsymbol{Og'_1} := \frac{r_{t'}}{l'_t}\cdot\int_0^{2\cdot\beta'_0} z_{0t'}(\alpha_{t'})\,d\alpha_{t'} \qquad \boldsymbol{Og'_2} := \frac{2\cdot r_{t'}}{{l'_t}^2}\cdot\int_0^{2\cdot\beta'_0} r_{t'}\cdot\alpha_{t'}\cdot z_{0t'}(\alpha_{t'})\,d\alpha_{t'}$$

$$\boldsymbol{\Delta_{at'}}(\theta) := \frac{1}{L_t}\cdot\left(l'_t\cdot f_{t'}(\theta,0)\cdot\boldsymbol{Og'_1} + f'_{t'}(\theta,0)\cdot\frac{{l'_t}^2}{2\cdot r_{t'}}\cdot\boldsymbol{Og'_2}\right) \qquad \boldsymbol{\Delta_{at'}}(\theta_0) = 0.013 + 0.015\mathrm{i}\,mm$$

Contribution du spiral entier

$$\boldsymbol{\Delta_1}(\theta) := \boldsymbol{\Delta_t}(\theta) + \boldsymbol{\Delta_s}(\theta) + \boldsymbol{\Delta_{t'}}(\theta) \qquad \boldsymbol{\Delta_1}(\theta_0) = 0.014 + 8.409\mathrm{i}\times 10^{-3}\,mm$$

$$u_1(\theta) := Re(\boldsymbol{\Delta_1}(\theta)) \qquad v_1(\theta) := Im(\boldsymbol{\Delta_1}(\theta)) \qquad u_1(\theta_0) = 0.014\,mm \qquad v_1(\theta_0) = 8.409\times 10^{-3}\,mm$$

Approximation

$$\boldsymbol{\Delta_a}(\theta) := \boldsymbol{\Delta_{at}}(\theta) + \boldsymbol{\Delta_{as}}(\theta) + \boldsymbol{\Delta_{at'}}(\theta) \qquad \boldsymbol{\Delta_a}(\theta_0) = 0.011 + 3.685\mathrm{i}\times 10^{-3}\,mm$$

Calcul des réactions

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\pi} x_{0t1}(\alpha_t)^2\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^{0} x_{0t2}(\beta_t)^2\cdot r_{t2}\,d\beta_t\right)$$

$$p2_{0s} := p2_{0s} + \frac{1}{L_t}\cdot\int_0^{2\cdot\beta'_0} x_{0t'}(\alpha_{t'})^2\cdot r_{t'}\,d\alpha_{t'} \qquad p2_{0s} = 1.403\,mm^2$$

$$q2_{0s} := \frac{1}{L_t} \cdot \left[\int_{\pi}^{\pi+\psi_0} y_{0s}(\alpha)^2 \cdot r_s(\alpha)\, d\alpha + \int_0^{\pi} y_{0t1}(\alpha_t)^2 \cdot r_{t1}\, d\alpha_t + \left(\int_{-\beta_0}^{0} y_{0t2}(\beta_t)^2 \cdot r_{t2}\, d\beta_t \right) \right]$$

$$q2_{0s} := q2_{0s} + \frac{1}{L_t} \cdot \int_0^{2\cdot\beta'_0} y_{0t'}(\alpha_{t'})^2 \cdot r_{t'}\, d\alpha_{t'} \qquad q2_{0s} = 1.391\, mm^2$$

$$k_{0s} := \frac{1}{L_t} \cdot \left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha) \cdot y_{0s}(\alpha) \cdot r_s(\alpha)\, d\alpha + \int_0^{\pi} x_{0t1}(\alpha_t) \cdot y_{0t1}(\alpha_t) \cdot r_{t1}\, d\alpha_t + \int_{-\beta_0}^{0} x_{0t2}(\beta_t) \cdot y_{0t2}(\beta_t) \cdot r_{t2}\, d\beta_t \right)$$

$$k_{0s} := k_{0s} + \frac{1}{L_t} \cdot \int_0^{2\cdot\beta'_0} x_{0t'}(\alpha_{t'}) \cdot y_{0t'}(\alpha_{t'}) \cdot r_{t'}\, d\alpha_{t'} \qquad k_{0s} = -0.013\, mm^2$$

$$\mathbf{S_0} := \frac{L}{E \cdot I_{33}} \cdot \begin{pmatrix} q2_{0s} & -k_{0s} \\ -k_{0s} & p2_{0s} \end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1} \cdot \begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} 6.405 \times 10^{-6} \\ 3.908 \times 10^{-6} \end{pmatrix} N$$

$$\left| \mathbf{R'}(\theta_0) \right| = 7.503 \times 10^{-6}\, N$$

Approximations

$$\sigma 2 := \frac{1}{L_t} \cdot \left[\int_{\pi}^{\pi+\psi_0} \left(\left| z_{0s}(\alpha) \right| \right)^2 \cdot r_s(\alpha)\, d\alpha + \int_0^{\pi} \left(\left| z_{0t1}(\alpha_t) \right| \right)^2 \cdot r_{t1}\, d\alpha_t + \int_{-\beta_0}^{0} \left(\left| z_{0t2}(\beta_t) \right| \right)^2 \cdot r_{t2}\, d\beta_t \right]$$

$$\sigma 2 := \sigma 2 + \frac{1}{L_t} \cdot \int_0^{2\cdot\beta'_0} \left(\left| z_{0t'}(\alpha_{t'}) \right| \right)^2 \cdot r_{t'}\, d\alpha_{t'} \qquad \sigma 2 = 2.794\, mm^2$$

$$R'(\theta) := \frac{E \cdot I_{33}}{L} \cdot \frac{2}{\sigma 2} \cdot \begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} 6.411 \times 10^{-6} \\ 3.983 \times 10^{-6} \end{pmatrix} N \qquad \left| R'(\theta_0) \right| = 7.547 \times 10^{-6}\, N$$

Perturbation de période - spiral non déformé en position de repos

$$X(\theta) := \frac{\left(\left| \mathbf{\Delta_1}(\theta) \right| \right)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta} X(\theta) \qquad Delta(\theta_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2\cdot\pi} \gamma(\theta_0 \cdot \cos(\varphi)) \cdot \cos(\varphi)\, d\varphi$$

$$\mu(\theta_0) := -86400 \cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = 0.976} \qquad \boxed{\mu(180 \cdot deg) = 0.424}$$

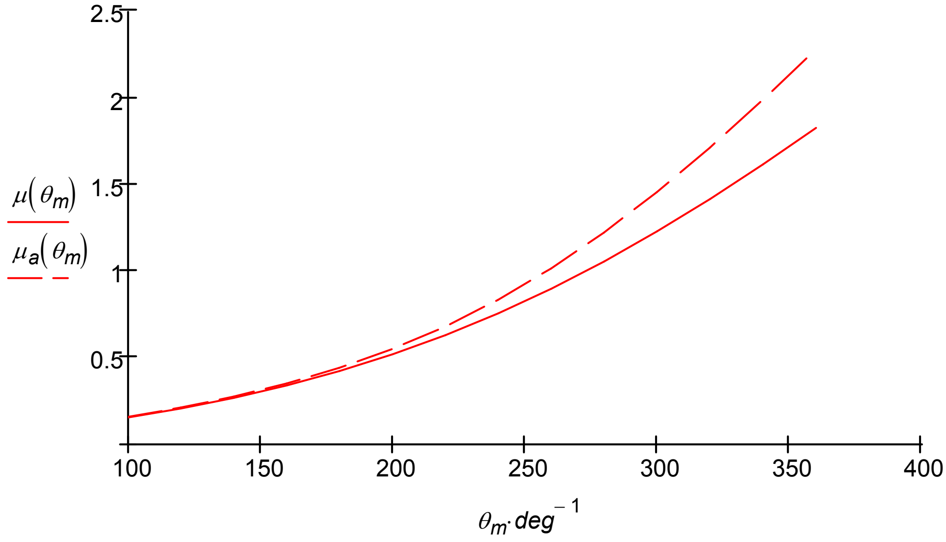
$$X(\theta) := \frac{\left(\left| \mathbf{\Delta_a}(\theta) \right| \right)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta} X(\theta) \qquad \delta_a(\theta_0) := \frac{-1}{2 \cdot \pi \cdot \theta_0} \cdot \int_0^{2\cdot\pi} \gamma(\theta_0 \cdot \cos(\varphi)) \cdot \cos(\varphi)\, d\varphi$$

$$\mu_a(\theta_0) := -86400 \cdot \delta_a(\theta_0) \qquad \boxed{\mu_a(\theta_0) = 1.118} \qquad \boxed{\mu_a(180 \cdot deg) = 0.441}$$

Correction du moment d'inertie du balancier

$$C := E \cdot \frac{I_{33}}{L_t} \qquad \Delta J_b := \frac{C}{\omega_0^{\,2}} - J_b \qquad \Delta J_b = -0.471\, mg \cdot cm^2 \qquad \frac{\Delta J_b}{J_b} = -4.708\,\%$$

$\theta_m := 100 \cdot deg, 120 \cdot deg .. 360 \cdot deg$



Référence :E:\Résonateur (TA)\Le spiral plat\SP avec CT_externe et interne - Approx de Haag.mcd(R)

$\theta_m := 100 \cdot deg, 120 \cdot deg .. 360 \cdot deg$